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Effect of retardation on the reflectance properties of the metallic Fibonacci quasi-superlattice

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Abstract. Based on the hydrodynamic model and the transfer-matrix method, we have re-examined the reflection properties of the metallic Fibonacci quasi-superlattice by taking account of the retardation effect on the systems. For the normal-incidence s-polarised soft x-rays and extreme ultraviolet, we find that the self-similar reflecting spectrum will be restrained on increasing the retardation, but for the higher-frequency region or at the smaller grazing angle, the self-similarity will still exist for the lower-generation quasi-superlattices.

1. Introduction

In the past few years, many scientists have become very interested in quasi-superlattices [1–13]. This is because, since the experimental observation of icosahedral symmetry in a quenched Al–Mn alloy [14], a wealth of fascinating physical information has been revealed both experimentally and theoretically for the quasi-periodic systems. For references, the recently published extensive review by Steinhard and Ostlund [15] should be consulted. In our recent paper [16] (hereafter referred to as I), a new transfer-matrix method, which was first presented by Xue and Tsai [17] and was later developed by Feng *et al* [18], was used for the calculation of the reflectivity of TE waves from a metallic Fibonacci quasi-superlattice (MFQSL). We found the very interesting phenomenon that, for the case of s-polarised soft x-rays and extreme ultraviolet, the reflecting spectrum pattern has self-similar properties around numerous scaling points with increasing the generation number. Also, we found that, because of the remodelling of the reflection curves as a function of frequency, some new peaks move to higher-frequency positions, which stimulates interest in the study and fabrication of soft x-ray and extreme-ultraviolet reflectors. One of the shortcomings of I is that, when the electron system in the MFQSL is considered as an ideal plasma liquid, the effect of retardation on the reflection was neglected; therefore the numerical results could not predict the real case quantitatively. In this paper, we shall improve the previous theory by including the retardation effect on the MFQSL system. For details of the model of the MFQSL system and relevant parameters, we refer the reader to I and [2].

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2. General formalism for the reflectivity

The MFQSL system is generated by two elementary seeds, blocks L and S, mapping the mathematical rule in the Fibonacci sequence, i.e.

$$S_1 = \{L\}, S_2 = \{LS\}, S_3 = (LSL\}, \ldots, S_n = S_{n-1}S_{n-2}. \tag{1}$$

Block L in this case is composed of metal A of thickness d_A and metal B of thickness d_{BL}, while for block S the only difference from block L is that the metal B in block S has a smaller thickness d_{BS}. As adopted in I, the ratio of the thickness of the two elementary blocks is just the inverse of the golden mean:

$$\Lambda \equiv (d_A + d_{BS})/(d_A + d_{BL}) = (\sqrt{5} - 1)/2. \tag{2}$$

To obtain the reflectivity for the s-polarised soft x-rays and extreme ultraviolet, we start from the general TE wave equation [16] for the MFQSL system,

$$\mathrm{d}^2E_y/\mathrm{d}z^2 + [(\omega/c)^2\varepsilon(\omega) - q^2]E_y = 0 \tag{3}$$

where q is the x-component wavevector, which is defined as a function of the incident angle θ:

$$q = (\omega/c)\sin\theta. \tag{4}$$

In I, the dielectric function $\varepsilon(\omega)$ in equation (3) is approximated in the form for the ideal case with no retardation:

$$\varepsilon_1(\omega) = 1 - \omega_P^2/\omega^2. \tag{5}$$

To take account of the retardation effect on the reflection, we choose the model dielectric function in the form of the Drude local dielectric function for the MFQSL system:

$$\varepsilon_D(\omega) = 1 - \omega_P^2/(\omega^2 + \mathrm{i}\omega/\tau) \tag{6}$$

where τ is the electric relaxation time in the metal layers.

We note that all the self-contained equations and the transfer matrices derived in I are still valid for the present calculation except that the wavevector in the metallic layers is changed to the complex form

$$k_\mu = \alpha_\mu + \mathrm{i}\beta_\mu \tag{7}$$

$$\alpha_\mu = \sqrt{\mathcal{A}_\mu}\,\{\tfrac{1}{2}[(1 + (\mathcal{B}_\mu/\mathcal{A}_\mu)^2)^{1/2} + 1]\}^{1/2} \tag{8}$$

$$\beta_\mu = \sqrt{\mathcal{A}_\mu}\,\{\tfrac{1}{2}[(1 + (\mathcal{B}_\mu/\mathcal{A}_\mu)^2)^{1/2} - 1]\}^{1/2} \tag{9}$$

$$\mathcal{A}_\mu = k_0^2 - (\omega_{P\mu}/c)^2/(1 + 1/\omega^2\tau_\mu^2) \tag{10}$$

$$\mathcal{B}_\mu = (\omega_{P\mu}/c)^2/\omega\tau_\mu(1 + 1/\omega^2\tau_\mu^2) \tag{11}$$

where

$$k_0 = (\omega/c)\cos\theta \qquad \omega_{P\mu} = \sqrt{4\pi n_\mu e^2/m} \tag{12}$$

and μ takes the value A or B for the two different electron densities n_A and n_B in the metallic multilayers. The transfer matrices of the MFQSL system, as a consequence, also become complex matrices. Following the scheme presented in I and through direct mathematical calculation, we finally obtain the reflectivity for the nth generation MFQSL as follows:

$$R = |r|^2 \tag{13}$$

$$r = [\eta_1(k_A + k_0) - \eta_2(k_A - k_0)]/[\eta_2(k_A + k_0) - \eta_1(k_A - k_0)] \tag{14}$$

$$\eta_1 = \exp(\mathrm{i}\, 2k_A d_A)[w_{21}(k_B + k_0) + w_{22}(k_B - k_0)] \tag{15}$$

$$\eta_2 = w_{11}(k_B + k_0) + w_{12}(k_B - k_0) \tag{16}$$

where w_{ij} denotes the element of the matrix W, which is defined as

$$W \equiv \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \equiv \begin{cases} X_A^{-1} C_n C_1 \tilde{X}_{BL} & \text{when } n = \text{odd number} \\ X_A^{-1} C_n C_0 \tilde{X}_{BS} & \text{when } n = \text{even number.} \end{cases} \tag{17}$$

Here

$$X_\mu \equiv \begin{bmatrix} 1 & 1 \\ \mathrm{i}k_\mu & -\mathrm{i}k_\mu \end{bmatrix} \tag{18}$$

$$\tilde{X}_\mu \equiv \begin{bmatrix} \exp(-\mathrm{i}k_\mu d_\mu) & \exp(\mathrm{i}k_\mu d_\mu) \\ \mathrm{i}k_\mu \exp(-\mathrm{i}k_\mu d_\mu) & -\mathrm{i}k_\mu \exp(\mathrm{i}k_\mu d_\mu) \end{bmatrix} \qquad \mu \equiv \mathrm{A}, \mathrm{B}L, \mathrm{B}S \tag{19}$$

and the nth-order transfer matrix C_n is generated in agreement with the Fibonacci sequence

$$C_n = C_{n-1} C_{n-2} \tag{20}$$

with the two initial generating seeds, corresponding to the two elementary units of the metallic multilayers, as defined in I:

$$C_0 = M_{BS} M_A \qquad C_1 = M_{BL} M_A \tag{21}$$

$$M_\mu \equiv \begin{bmatrix} \cos k_\mu d_\mu & -(\sin k_\mu d_\mu)/k_\mu \\ k_\mu \sin k_\mu d_\mu & \cos k_\mu d_\mu \end{bmatrix} \qquad \mu \equiv \mathrm{A}, \mathrm{B}L, \mathrm{B}S. \tag{22}$$

3. Numerical results and conclusions

In order to compare these results with the previous theoretical results, we take the relevant parameters the same as in I. We still choose the thicknesses of the metal layers as $d_A = 100$ Å and $d_{BL} = 200$ Å. For the metallic electron densities, we take $n_A = 8.63 \times 10^{23}$ cm^{-3} and $n_B = 18.1 \times 10^{23}$ cm^{-3}. In the frequency range from 0.75 ω_{PB} to 2.75 ω_{PB}, we plot the two calculated reflectance curves for normal incidence in figure 1. The curve in figure 1(b) is for the case $\tau = 100/\omega_P$, and the curve in figure 1(a) is for the limiting case, $\tau = \infty$, in which the effect of the retardation is neglected, which is just the same as the case in I.

From our numerical results, we find that, when the retardation is taken into account, the maxima of the reflection are reduced monotonically. The self-similar pattern of the curves is thus restrained strongly in the low-frequency range with increasing generation number. This can be easily understood for we know that, when the electric relaxation

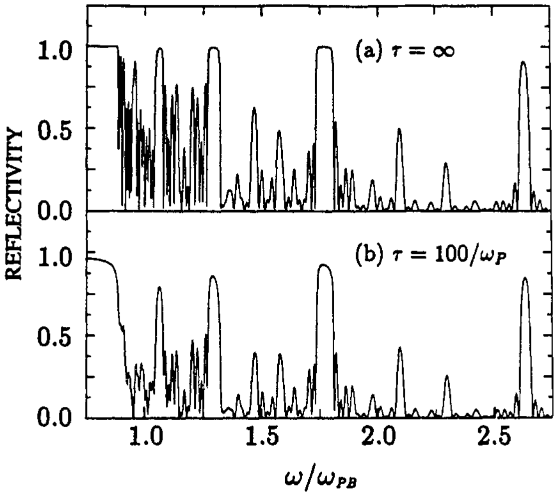

Figure 1. Reflectivities for normal incidence on the ninth-generation MFQSL with 110 metal layers ($\omega_{PB} = 2.4 \times 10^{16}$ s^{-1}, $d_A = 100$ Å and $d_{BL} = 200$ Å): (*a*) ideal case without the retardation effect, $\tau = \infty$; (*b*) $\tau = 100/\omega_P$.

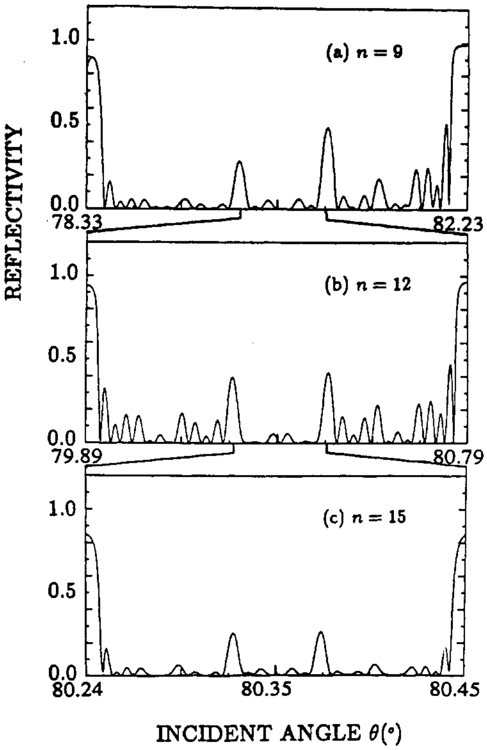

Figure 2. Reflectivities of the grazing incidence on the three generation MFQSL systems (*a*) S_9, (*b*) S_{12} and (*c*) S_{15} for an incident wavelength λ of 60 Å. The other relevant parameters are the same as in figure 1.

time τ approaches infinity, which means that there is no damping in the system, all of the metal layers will give contributions to the phase superposition because of reflection, which enhances the self-similarity but, when τ is introduced into the dielectric function, the consequent damping effect restricts the reflecting contribution from the metal layers far from the surface of the MFQSL. So, when we increase the generation number, which means adding more metal layers, the fine structure of the self-similar pattern of the reflectivity will gradually disappear at the ideal scaling points owing to the limiting contribution from the extra layers.

Fortunately, we find that, for the high-frequency region or for grazing incidence, the effect of retardation can be ignored owing to the form of the dielectric function that we have chosen. This is also true in real experiments. Figure 2 shows the rescaled reflectance curves for three generation numbers $n = 9$, 12 and 15 around an incidence angle of 80.35°, which shows that the self-similarity still exists for large incidence angles. From our present theoretical calculations, we find that for the finite generation numbers ($n \leqslant 18$), the six-circle property [5] around the fixed points for the reflectivities is approximately of the following form:

$$R_{n+6}[\Lambda^6(\theta - \theta_0)] = R_n(\theta - \theta_0). \tag{23}$$

This equation suggests a simple way to test the predicted scaling results.

As the reflectivity for the grazing incidence of higher frequencies will be only slightly affected by the effect of retardation, the aperiodic design of MFQSL is a stimulating way to enhance reflectivities for soft x-rays and extreme ultraviolet.

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